جَامعة
المَنارة
MANARA UNIVERSITY

Finite Element Methods

Week No-04

Introduction to Matrix Structural Analysis

- Classical Methods vs. Matrix Methods.
- Planar Frame Member, Nodal Displacements & Nodal Forces in Local Coordinates
- Truss Analysis by Stiffness Matrix Method
    - Truss Member Stiffness Relations in Local Coordinates
    - Member Stiffness Relations in Global Coordinates
    - Structure Stiffness Relations
    - Procedure for Analysis
- Frame & Beam Analysis by Stiffness Matrix Method
    - Analytical Model for Planar Frame Structure
    - Global & Local Coordinate Systems
    - Member Stiffness Relations in Local Coordinates
    - Stiffness Matrix of 2D Frame Member in Global Coordinates
    - Member Stiffness Relations in Global Coordinates
    - Structure Stiffness Relations

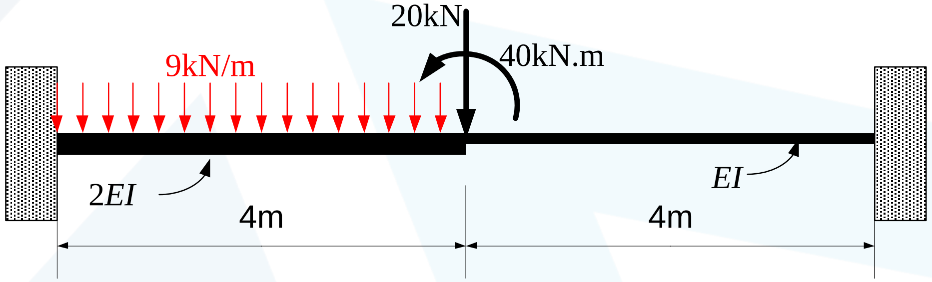
Classical Methods vs Matrix Methods

Classical Methods	Matrix Methods
Help to understand the structural behavior & the principles of structural Analysis	Simplify the overall picture of Structural Analysis
Time consuming for the analysis of large systems	Time saving as being computerized
Vary according to the structure type	Less varying

Example-02

For the given beam use the matrix method to:

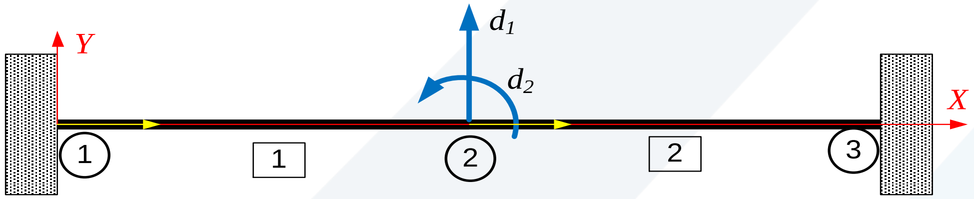
1. Construct the Analytical model & determine the global degrees of freedom.
2. For each member construct the global stiffness matrix.
3. Then establish the beam stiffness matrix & solve for the global degrees of freedom.
4. Finally Determine end forces in each member and the support reactions. (hand writing)



Solution:

1- Analytical Model & Degrees of Freedom:

From the Analytical model in the next Fig. a) the beam has only two d.o.f. d_1 & d_2: joint 2 global deflection and rotation.
b) Local coordinates are identical to the global ones:
$[T]=[T]^T=[I]$

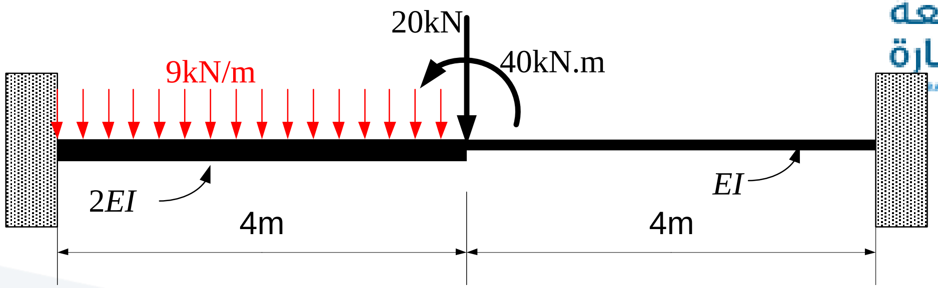


Analytical Model

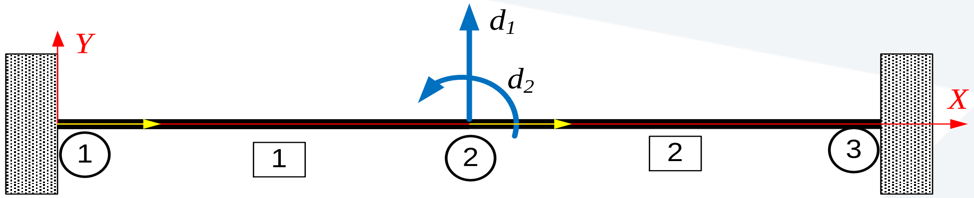
Example-02

Solution:

1- Analytical Model & Degrees of Freedom:



From the Analytical model in the next Fig. a) the beam has only two d.o.f. d_1 & d_2: joint 2 global deflection and rotation.
b) Local coordinates are identical to the global ones:
$[T]=[T]^T=[I]$



Analytical Model

2- Member Stiffness Matrices in Global Coo.:

Using the following 4×4 local stiffness matrix

$$[K]_m = \begin{bmatrix} 12EI/L^3 & 6EI/L^2 & -12EI/L^3 & 6EI/L^2 \\ 6EI/L^2 & 4EI/L & -6EI/L^2 & 2EI/L \\ -12EI/L^3 & -6EI/L^2 & 12EI/L^3 & -6EI/L^2 \\ 6EI/L^2 & 2EI/L & -6EI/L^2 & 4EI/L \end{bmatrix}$$

a – Member1: $[k]_1 = [K]_1$

$$\begin{array}{rc} & \begin{array}{cccc} 0 & 0 & 1 & 2 \end{array} \\ [K]_1 = EI & \left[\begin{array}{cccc} 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.75 & 2 \end{array}\right]\begin{array}{c} 0 \\ 0 \\ 1 \\ 2 \end{array} \end{array}$$

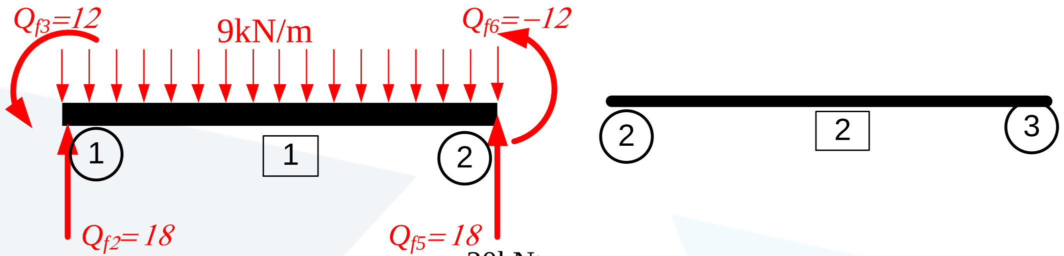
b – Member2: $[k_2] = [K_2]$

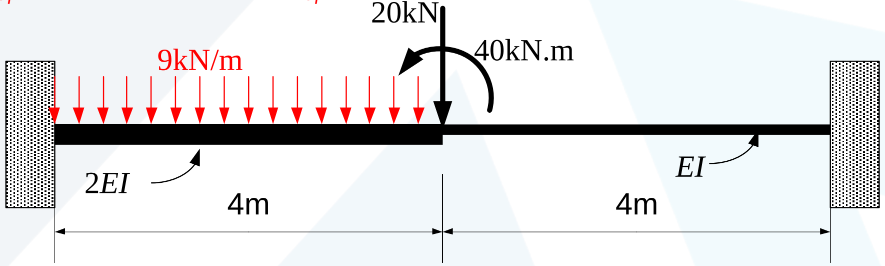
$$\begin{array}{rc} & \begin{array}{cccc} 1 & 2 & 0 & 0 \end{array} \\ [K]_2 = EI & \left[\begin{array}{cccc} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{array}\right]\begin{array}{c} 1 \\ 2 \\ 0 \\ 0 \end{array} \end{array}$$

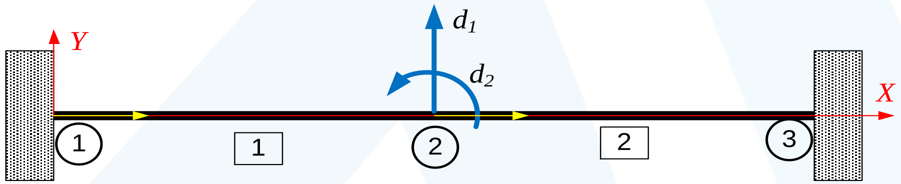
3 – Structure (or Beam) Stiffness Matrix S:

As [T]=[T]^T=I, the beam member global stiffness equation is: $\{F\}=[K]\{v\}+\{F_f\}$, and from the fixed end separated members:

$$\{F_f\}_1 = \begin{Bmatrix} 18 \\ 12 \\ 18 \\ -12 \end{Bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} \ \& \ \{F_f\}_2 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$







a- Member Matrices assembling:

From above and from the given data, the global beam stiffness equation is: $\{P\}=[S]\{d\}+\{P_f\}$. In detailed matrix form, this is written as

$$\{P\}=[S]\{d\}+\{P_f\}$$

$$\begin{Bmatrix} -20 \\ 40 \end{Bmatrix} = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} + \begin{Bmatrix} 18 \\ -12 \end{Bmatrix}$$

b- Solving for the global degrees of freedom:

$$\begin{Bmatrix} -38 \\ 52 \end{Bmatrix} = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \Rightarrow \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -61.09 \\ 9.697 \end{Bmatrix}$$

4 – Member End Displacements & End Forces and Reactions:

a – Member1:

Global (& Local) Displacement Vector:

$$\{v\}_1=\{u\}_1=\begin{bmatrix}v_1\\v_2\\v_3\\v_4\end{bmatrix}\begin{matrix}0\\0\\1\\2\end{matrix}=\begin{bmatrix}0\\0\\d_1\\d_2\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}0\\0\\-61.09\\9.697\end{bmatrix}$$

Global (& Local) Force Vector

$$\{F\}_1=[K]_1\{v\}_1+\{F_f\}_1=EI\begin{bmatrix}0.375&0.75&-0.375&0.75\\0.75&2&-0.75&1\\-0.375&-0.75&0.375&-0.75\\0.75&1&-0.75&2\end{bmatrix}\left(\frac{1}{EI}\begin{Bmatrix}0\\0\\-61.09\\9.697\end{Bmatrix}\right)+\begin{Bmatrix}18\\12\\18\\-12\end{Bmatrix}=\begin{Bmatrix}48.18\text{kN}\\67.51\text{kN.m}\\-12.18\text{kN}\\53.21\text{kN.m}\end{Bmatrix}$$

b – Member2:

Global (& Local) Displacement Vector:

$$\{v\}_2=\{u\}_2=\begin{bmatrix}v_1\\v_2\\v_3\\v_4\end{bmatrix}\begin{matrix}1\\2\\0\\0\end{matrix}=\begin{bmatrix}d_1\\d_2\\0\\0\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}-61.09\\9.697\\0\\0\end{bmatrix}$$

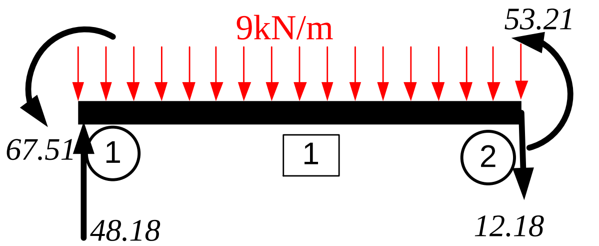
Global (& Local) Force Vector

$$\{F\}_2=[K]_2\{v\}_2+0=EI\begin{bmatrix}0.1875&0.375&-0.1875&0.375\\0.375&1&-0.375&0.5\\-0.1875&-0.375&0.1875&-0.375\\0.375&0.5&-0.375&1\end{bmatrix}\left(\frac{1}{EI}\begin{Bmatrix}-61.09\\9.697\\0\\0\end{Bmatrix}\right)=\begin{Bmatrix}-7.818\text{kN}\\-13.21\text{kN.m}\\7.818\text{kN}\\-18.06\text{kN.m}\end{Bmatrix}$$

C– Reactions:

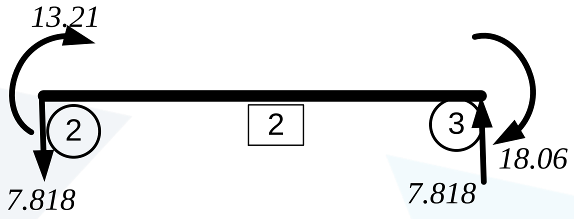
From the global force vector of member1, the FBD of this member is drawn

$$\{F\}_1=\left\{\begin{array}{c|c} 48.18\text{kN} & 0\\ 67.51\text{kN.m} & 0\\ -12.18\text{kN} & 1\\ 53.21\text{kN.m} & 2\end{array}\right.$$



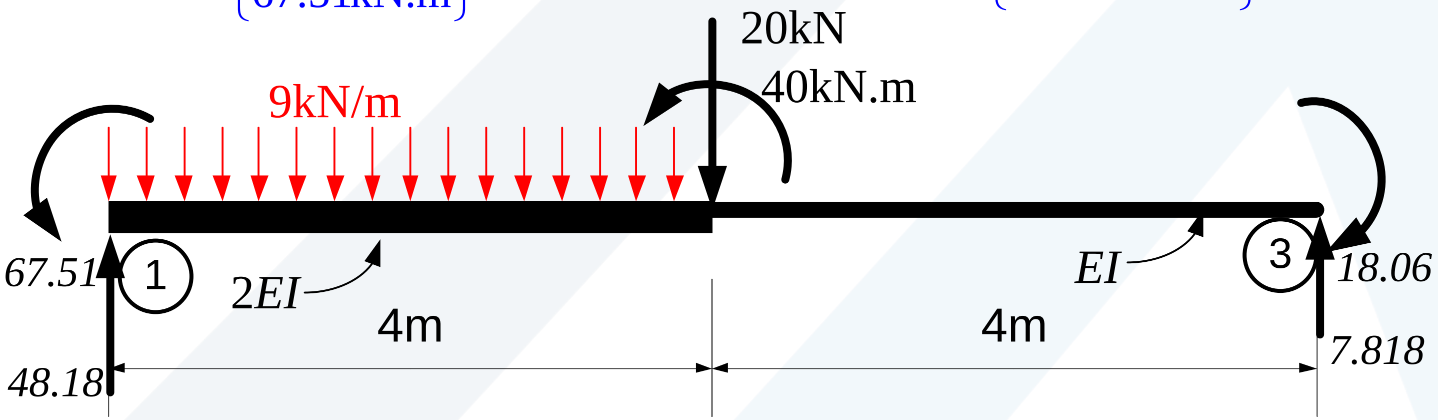
From the global force vector of member2, the FBD of this member is drawn

$$\{F\}_2=\left\{\begin{array}{c|c} -7.818\text{kN} & 1\\ -13.21\text{kN.m} & 2\\ 7.818\text{kN} & 0\\ -18.06\text{kN.m} & 0\end{array}\right.$$



From the above two vectors and the two FBDs, the reactions at joints 1 & 2 are

$$\{R\}_1=\left\{\begin{array}{c} 48.18\text{kN}\\ 67.51\text{kN.m}\end{array}\right\} \qquad \{R\}_2=\left\{\begin{array}{c} 7.818\text{kN}\\ -18.06\text{kN.m}\end{array}\right\}$$



Stiffness Matrix of 2D Frame Member in Global Coordinates

The stiffness matrix **k** (6×6) of the 2D frame member in local coor. written as

$$\mathbf{k} = \frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

$$\mathbf{Q} = \mathbf{k}\mathbf{u} + \mathbf{Q}_f$$

can be separated into the two following matrices

The stiffness matrix **k** (4×4) of the 2D beam member in local coor.

$$\mathbf{k} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

& the stiffness matrix **k** (2×2) of 2D truss member in local coor.

$$\mathbf{k} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

The beam member matrix can be used in global coor.
But the frame & truss matrices must be transformed from local to global coordinates using **T & T**$^{\mathrm{T}}$.

Kinematic Transformation from global to local coordinates

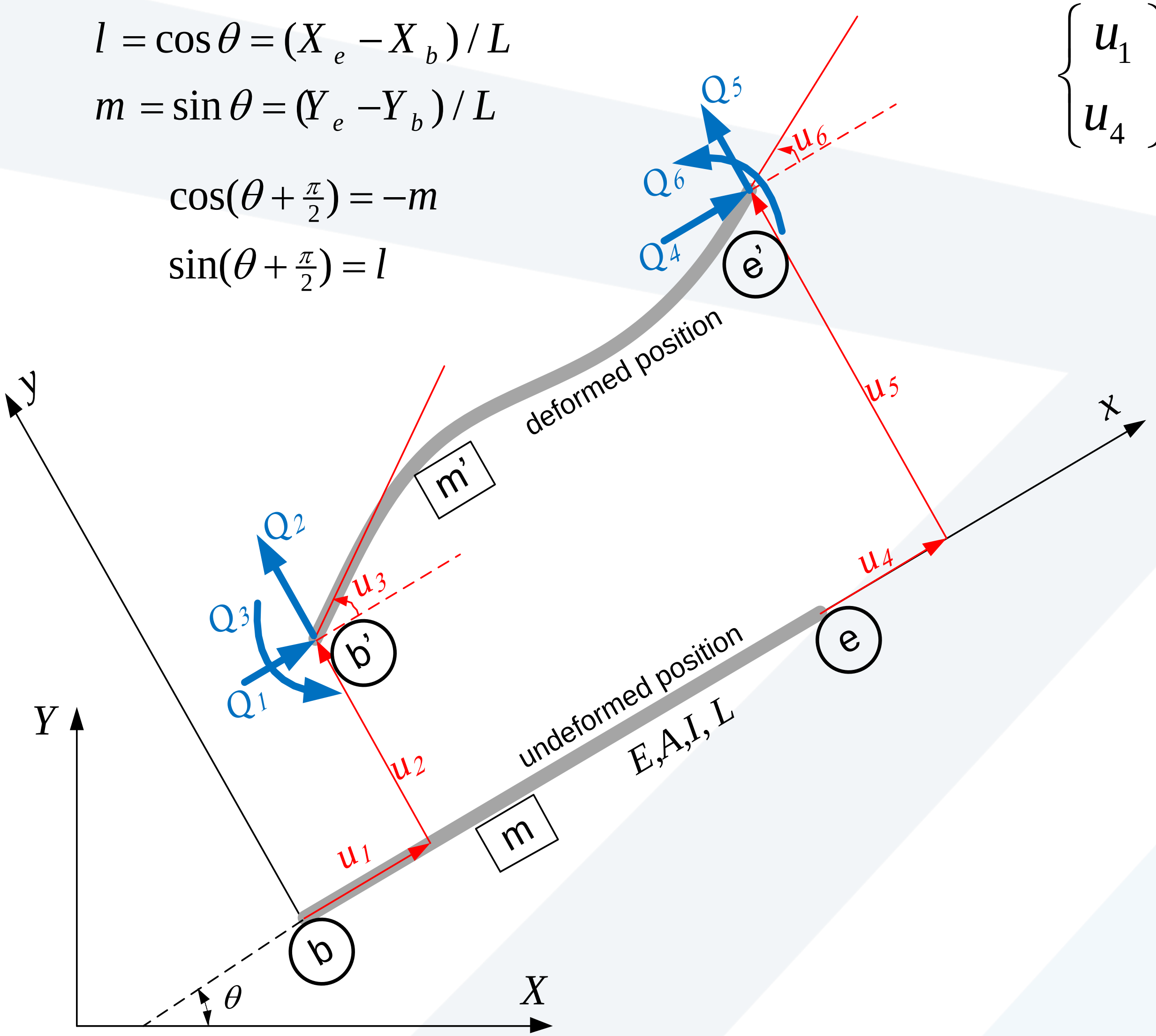
$$L = \sqrt{(X_e - X_b)^2 + (Y_e - Y_b)^2}$$

$$l = \cos\theta = (X_e - X_b)/L$$

$$m = \sin\theta = (Y_e - Y_b)/L$$

$$\cos(\theta + \tfrac{\pi}{2}) = -m$$

$$\sin(\theta + \tfrac{\pi}{2}) = l$$



$$\begin{Bmatrix} u_1 \\ u_4 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix}$$ for truss member

for frame member $\mathbf{u}_{6\times1} = \mathbf{T}_{6\times6}\mathbf{v}_{6\times1}$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{bmatrix} l & m & 0 & 0 & 0 & 0 \\ -m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & 0 \\ 0 & 0 & 0 & -m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{Bmatrix}$$

Static Transformation from local to global coordinates

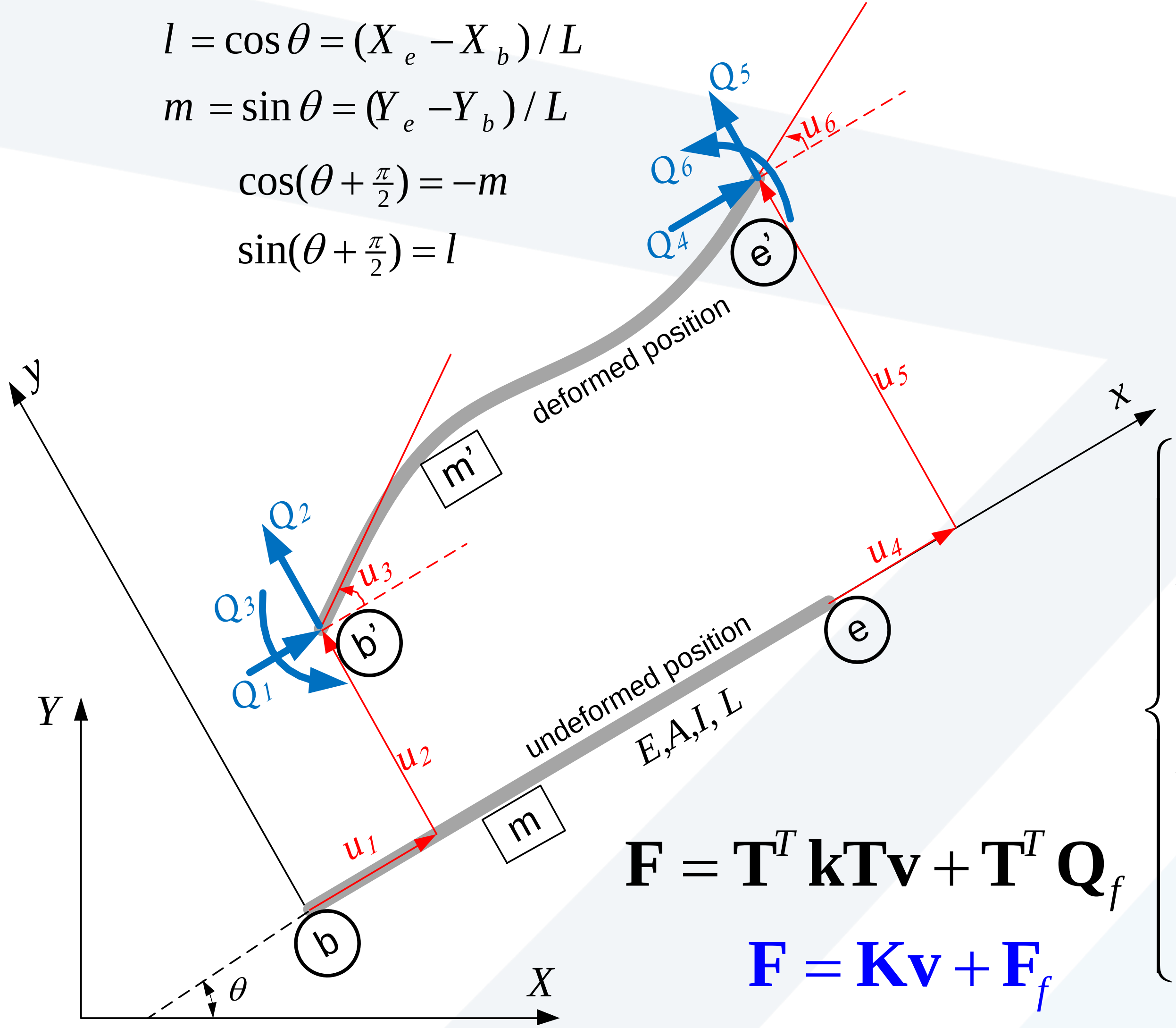
$$L = \sqrt{(X_e - X_b)^2 + (Y_e - Y_b)^2}$$

$$l = \cos\theta = (X_e - X_b)/L$$

$$m = \sin\theta = (Y_e - Y_b)/L$$

$$\cos(\theta + \tfrac{\pi}{2}) = -m$$

$$\sin(\theta + \tfrac{\pi}{2}) = l$$



$$\mathbf{F} = \mathbf{T}^T \mathbf{k} \mathbf{T} \mathbf{v} + \mathbf{T}^T \mathbf{Q}_f$$

$$\mathbf{F} = \mathbf{K} \mathbf{v} + \mathbf{F}_f$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_4 \end{Bmatrix}$$

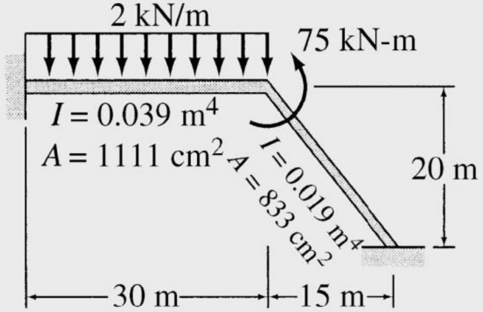
for truss member

for frame member $\mathbf{F}_{6\times1} = \mathbf{T}^{\mathrm{T}}_{6\times6} \mathbf{Q}_{6\times1}$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \begin{bmatrix} l & -m & 0 & 0 & 0 & 0 \\ m & l & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l & -m & 0 \\ 0 & 0 & 0 & m & l & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{Bmatrix}$$

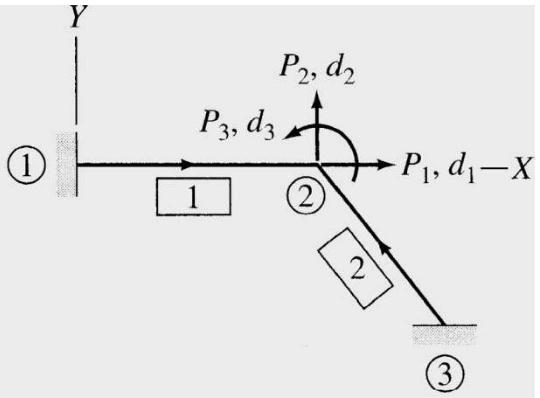
Ex. Determine the reactions and the member end forces for the frame shown in Fig. (a) by using the matrix stiffness method.

$$\mathbf{K} = \mathbf{T}^T \mathbf{k} \mathbf{T} \quad \& \quad \mathbf{F}_f = \mathbf{T}^T \mathbf{Q}_f$$



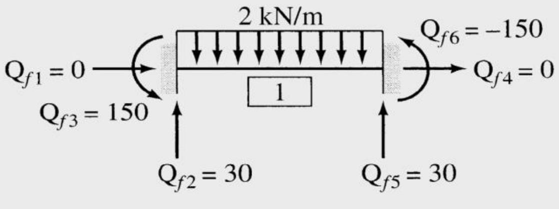
E=……

(a) Frame



(b) Analytical Model

$$\mathbf{K}_1 = \mathbf{k}_1 = \frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

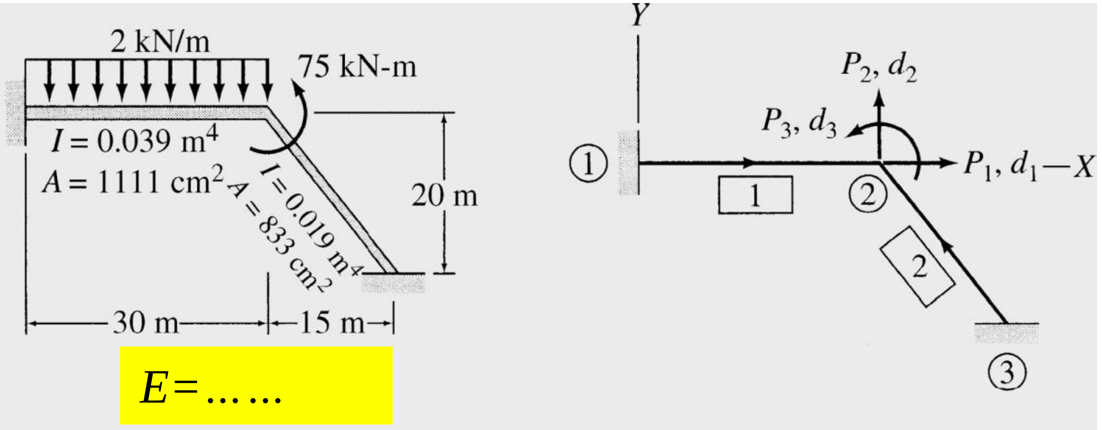


(c) Member Fixed-End Forces

$$\mathbf{F}_{f1} = \mathbf{Q}_{f1} = \begin{array}{c|c} \left[\text{[redacted]}\right] & \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$\mathbf{K}_1 = \mathbf{k}_1 = \begin{array}{cccccc|c} 0 & 0 & 0 & 1 & 2 & 3 & \\ \hline \multicolumn{6}{c|}{\text{[redacted]}} & 0 \\ \multicolumn{6}{c|}{\text{[redacted]}} & 0 \\ \multicolumn{6}{c|}{\text{[redacted]}} & 0 \\ \multicolumn{6}{c|}{\text{[redacted]}} & 1 \\ \multicolumn{6}{c|}{\text{[redacted]}} & 2 \\ \multicolumn{6}{c|}{\text{[redacted]}} & 3 \end{array}$$

$$\mathbf{k}_2=\frac{EI}{L^3}\begin{bmatrix}\frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0\\ 0 & 12 & 6L & 0 & -12 & 6L\\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2\\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0\\ 0 & -12 & -6L & 0 & 12 & -6L\\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2\end{bmatrix}$$



(a) Frame

(b) Analytical Model

$$\mathbf{k}_2=\begin{bmatrix}13{,}920 & 0 & 0 & -13{,}920 & 0 & 0\\ 0 & 61.87 & 773.33 & 0 & -61.87 & 773.33\\ 0 & 773.33 & 12{,}888.89 & 0 & -773.33 & 6{,}444.44\\ -13{,}920 & 0 & 0 & 13{,}920 & 0 & 0\\ 0 & -61.87 & -773.33 & 0 & 61.87 & -773.33\\ 0 & 773.33 & 6{,}444.44 & 0 & -773.33 & 12{,}888.89\end{bmatrix}$$

$$\mathbf{T}_2=\begin{bmatrix}-0.6 & 0.8 & 0 & 0 & 0 & 0\\ -0.8 & -0.6 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & -0.6 & 0.8 & 0\\ 0 & 0 & 0 & -0.8 & -0.6 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

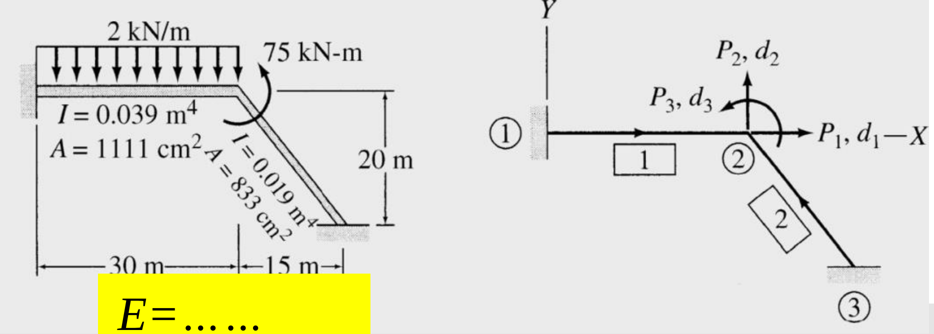
$$\mathbf{K}_2=\begin{array}{c}\begin{array}{cccccc}0 & 0 & 0 & 1 & 2 & 3\end{array}\\ \left[\begin{array}{cccccc}5{,}050.8 & -6{,}651.9 & -618.67 & -5{,}050.8 & 6{,}651.9 & -618.67\\ -6{,}651.9 & 8{,}931.07 & -464 & 6{,}651.9 & -8{,}931.07 & -464\\ -618.67 & -464 & 12{,}888.89 & 618.67 & 464 & 6{,}444.44\\ -5{,}050.8 & 6{,}651.9 & 618.67 & 5{,}050.8 & -6{,}651.9 & 618.67\\ 6{,}651.9 & -8{,}931.07 & 464 & -6{,}651.9 & 8{,}931.07 & 464\\ -618.67 & -464 & 6{,}444.44 & 618.67 & 464 & 12{,}888.89\end{array}\right]\end{array}\begin{array}{c}\\ 0\\ 0\\ 0\\ 1\\ 2\\ 3\end{array}$$

$$\mathbf{K}_1 = \mathbf{k}_1 = \begin{array}{c} \begin{array}{cccccc} 0 & 0 & 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|ccc} 15{,}466.67 & 0 & 0 & -15{,}466.67 & 0 & 0 \\ 0 & 71.6 & 1{,}074.07 & 0 & -71.6 & 1{,}074.07 \\ 0 & 1{,}074.07 & 21{,}481.48 & 0 & -1{,}074.07 & 10{,}740.74 \\ \hline -15{,}466.67 & 0 & 0 & 15{,}466.67 & 0 & 0 \\ 0 & -71.6 & -1{,}074.07 & 0 & 71.6 & -1{,}074.07 \\ 0 & 1{,}074.07 & 10{,}740.74 & 0 & -1{,}074.07 & 21{,}481.48 \end{array}\right] \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

$$\mathbf{K}_2 = \begin{array}{c} \begin{array}{cccccc} 0 & 0 & 0 & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|ccc} 5{,}050.8 & -6{,}651.9 & -618.67 & -5{,}050.8 & 6{,}651.9 & -618.67 \\ -6{,}651.9 & 8{,}931.07 & -464 & 6{,}651.9 & -8{,}931.07 & -464 \\ -618.67 & -464 & 12{,}888.89 & 618.67 & 464 & 6{,}444.44 \\ \hline -5{,}050.8 & 6{,}651.9 & 618.67 & 5{,}050.8 & -6{,}651.9 & 618.67 \\ 6{,}651.9 & -8{,}931.07 & 464 & -6{,}651.9 & 8{,}931.07 & 464 \\ -618.67 & -464 & 6{,}444.44 & 618.67 & 464 & 12{,}888.89 \end{array}\right] \end{array} \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

The Frame stiffness matrix is

$$\begin{bmatrix} 20{,}517.47 & -6{,}651.9 & 618.67 \\ -6{,}651.9 & 9{,}002.67 & -610.07 \\ 618.67 & -610.07 & 34{,}370.37 \end{bmatrix}$$



$E = \ldots\ldots$

$$\mathbf{P} - \mathbf{P}_f = \mathbf{S}\mathbf{d},$$

$$\begin{bmatrix} 0 \\ 0 \\ 75 \end{bmatrix} - \begin{bmatrix} 0 \\ 30 \\ -150 \end{bmatrix} = \begin{bmatrix} 20{,}517.47 & -6{,}651.9 & 618.67 \\ -6{,}651.9 & 9{,}002.67 & -610.07 \\ 618.67 & -610.07 & 34{,}370.37 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ -30 \\ 225 \end{bmatrix} = \begin{bmatrix} 20{,}517.47 & -6{,}651.9 & 618.67 \\ -6{,}651.9 & 9{,}002.67 & -610.07 \\ 618.67 & -610.07 & 34{,}370.37 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} -0.00149\ \text{m} \\ -0.00399\ \text{m} \\ 0.0065\ \text{rad} \end{bmatrix}$$

Member End Displacements and End Forces

$$\mathbf{u}_1 = \mathbf{v}_1 = \begin{array}{c} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00149\ \text{m} \\ -0.00399\ \text{m} \\ 0.0065\ \text{rad} \end{bmatrix}$$

$$\mathbf{F}_1 = \mathbf{Q}_1 = \begin{bmatrix} 23.05\ \text{kN} \\ 37.27\ \text{kN} \\ 224.1\ \text{kN-m} \\ -23.05\ \text{kN} \\ 22.73\ \text{kN} \\ -6.08\ \text{kN-m} \end{bmatrix}$$

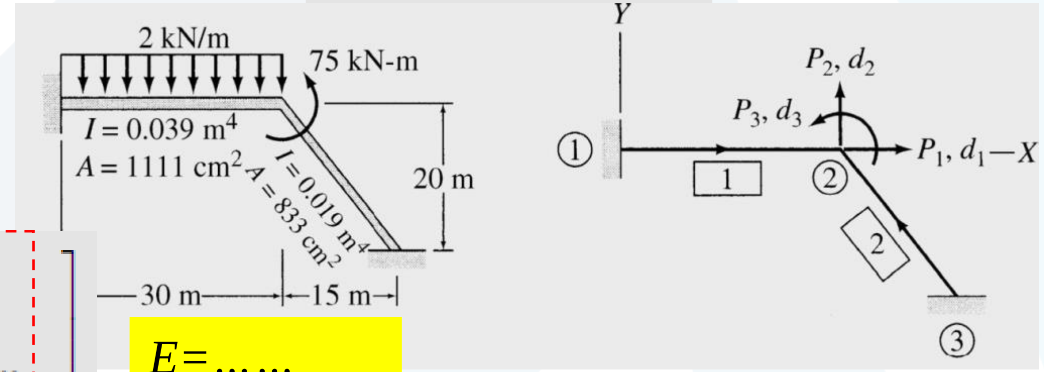
Member 2

$$\mathbf{k}_2 = \begin{bmatrix} 13{,}920 & 0 & 0 & -13{,}920 & 0 & 0 \\ 0 & 61.87 & 773.33 & 0 & -61.87 & 773.33 \\ 0 & 773.33 & 12{,}888.89 & 0 & -773.33 & 6{,}444.44 \\ -13{,}920 & 0 & 0 & 13{,}920 & 0 & 0 \\ 0 & -61.87 & -773.33 & 0 & 61.87 & -773.33 \\ 0 & 773.33 & 6{,}444.44 & 0 & -773.33 & 12{,}888.89 \end{bmatrix}$$

$$\mathbf{T}_2 = \begin{bmatrix} -0.6 & 0.8 & 0 & 0 & 0 & 0 \\ -0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6 & 0.8 & 0 \\ 0 & 0 & 0 & -0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{array}{c} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.00149\text{ m} \\ -0.00399\text{ m} \\ 0.0065\text{ rad} \end{bmatrix}$$

$$\mathbf{F}_2 = \begin{bmatrix} -23.04\text{ kN} \\ 22.71\text{ kN} \\ 39.12\text{ kN-m} \\ 23.04\text{ kN} \\ -22.71\text{ kN} \\ 81\text{ kN-m} \end{bmatrix} \qquad \mathbf{Q}_2 = \begin{bmatrix} 31.99\text{ kN} \\ 4.81\text{ kN} \\ 39.12\text{ kN-m} \\ -31.99\text{ kN} \\ -4.81\text{ kN} \\ 81\text{ kN-m} \end{bmatrix}$$



$E = \ldots\ldots$

Support Reactions

$$\mathbf{F}_1 = \mathbf{Q}_1 = \begin{bmatrix} \boxed{\begin{matrix} 23.05\text{ kN} \\ 37.27\text{ kN} \\ 224.1\text{ kN-m} \end{matrix}} \\ -23.05\text{ kN} \\ 22.73\text{ kN} \\ -6.08\text{ kN-m} \end{bmatrix} \qquad \mathbf{F}_2 = \begin{bmatrix} \boxed{\begin{matrix} -23.04\text{ kN} \\ 22.71\text{ kN} \\ 39.12\text{ kN-m} \end{matrix}} \\ 23.04\text{ kN} \\ -22.71\text{ kN} \\ 81\text{ kN-m} \end{bmatrix}$$

$$\mathbf{R}_{①} = \begin{bmatrix} 23.05\text{ kN} \\ 37.27\text{ kN} \\ 224.1\text{ kN-m} \end{bmatrix}, \qquad \mathbf{R}_{③} = \begin{bmatrix} -23.04\text{ kN} \\ 22.71\text{ kN} \\ 19.12\text{ kN-m} \end{bmatrix}$$